STOCK OPTION WARRANT ANALYSIS

David Atwood Fitch

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

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by

David Atwood Fitch

Thesis Advisor: P. M. Carrick

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David Atwood Fitch
Lieutenant, United States Navy
B.S., United States Naval Academy, 1966

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ABSTRACT

The following is an analysis of stock option warrants from the investor point of view. A survey of the literature presents the forms of analysis used to date. A model is called the Fitch Model, for reasons of humility, which will explain warrant value in terms of associated stock variability, yield, leverage, and potential common stock dilution. Time to expiration of the warrant is discounted by considering only warrants with more than seven years until expiration. The analysis also presents two other models, Kassouf's and a linear regression, as a basis for comparison. The conclusions are that the Kassouf model is both heteroscedastic and first order autocorrelated and could not support further analysis without modifying its structure. Of the two remaining models the linear model yields superior predictions as measured by its standard error. It is also felt that a more representative sample of warrants may significantly change the results given here.

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I. INTRODUCTION

Stock option warrants are a special type of investment vehicle. The low price of the stock option warrant relative to the associated common stock price and the call feature make warrants a controversial and often misunderstood method of investment. This speculative investment medium was instituted in this country by The American Power and Light Company. A senior security with common stock option warrants attached was issued in 1911. In doing so they created a new form of negotiable security which potentially could benefit both writer and buyer of the warrant.

Reasons for issuing convertible securities in general are usually hypothecated as a desire by management to raise common capital indirectly and to improve the market acceptance of a bond or preferred stock contract (Ref. 21). The other facet of warrant appeal concerns the investor who can now expand his portfolio to include this highly speculative security. This paper will address the question concerning warrant valuation in the market place as an investment medium.

The history of the theory of warrant pricing will first be discussed. Warrants have been examined extensively during the past ten years from the standpoint of risk, utility, general equilibrium, and naive empiricism by such as Samuelson (Ref. 23), Merton (Ref. 17), Black and Scholes (Ref. 4), and Sprenkle (Ref. 25). Although much work has been done, there still remain unanswered questions.

In order to develop the theory underlying the model which attempts to explain stock option warrant valuation an outline of ideas as pertains to this model will be presented. It is not intended to cover all aspects

contained in the literature but merely highlight those topics relevant to the development of the model.

A stock option warrant will first be defined by use of the Trans World Airlines (TWA) warrant. Relative movements of stock and warrant prices are illustrated by the stock/warrant diagram of TWA. This diagram points out limiting conditions along with a typical sample realization of stock and warrant prices. This basic stock/warrant diagram is the intuitive foundation for further analysis and the previous work of many as outlined in Chapter Four.

Although the stock price is easily the most important variable in explaining the associated price of the warrant, usually accounting for roughly ninety percent of the warrant price movement, the explanation of the residual error by means of other variables through a mathematical model is the thrust of this paper.

Among the other variables, time until warrant expiration and volatility of the stock seemed most worthy of analysis. From the historical development of warrant analysis the variable time will be seen to have been scrutinized empirically to some detail. This is done in both the Kassouf (Ref. 12) and the Miller (Ref. 18) analyses. It was therefore decided to concentrate upon the associated stock volatility as an endogenous variable. The final paradigm, the Fitch Model, will be compared to the Kassouf Model and, in keeping with Occam's razor, a simple linear model.

II. STOCK OPTION WARRANT DEFINED

A warrant is a contract which permits one to buy a share of a given common stock at a stipulated price called the exercise price, until a specified date occurs, the expiration date.

The TWA warrant is a good example. These warrants were originally attached to TWA's subordinated income debentures of 6 1/2% per annum of June 8, 1961, maturing June 1, 1978. The warrants were issued with each $100 debenture for purchase of 2.7 common shares at $20 per share from November 1, 1961 to June 1, 1965 and $22 thereafter to December 1, 1973.

The purchase price for the stock is payable either in cash or by surrender of bonds at par (without credit for accrued interest or dividends). The warrants are protected against dilution of the common stock, i.e., if the stock splits or stock dividends are distributed the warrant call feature will be adjusted to conform to the prior or original amount (Ref. 19).

Warrants are customarily attached to senior securities initially but on a specified date can be traded independently of the senior security. Warrants, at that time, are then priced according to the emotion of the market place. Typical features which would cause problems in the analysis are accurate measurement of warrant value with step up in exercise price, and accounting for exercise prices that could be settled by either cash or the principal value of a fixed-income instrument that may trade at other than par.

III. STOCK/WARRANT DIAGRAM

It is useful to think of warrant price movement as primarily a function of its associated stock price. It will be seen later that the stock price is the principle cause of warrant movement. This implicitly assumes that the stock price is known deterministically or can be described by a probability density function. The objective is to develope a descriptive model rather than a predictive model. Of course knowledge of the causes of past movements of warrant prices is valuable information in evaluating future potential.

If an investor knew exactly what a stock would be selling for at all times prior to expiration, he would know the true value of the warrant. The true value being what the warrant is actually worth if exercised, i.e., T.V. = MAX (0,S-E). For example, suppose the market conditions were as follows:

Stock Price:	S = 15.00
Warrant Price:	W = 5.00
Exercise Price:	E = 22.00

If S were known with certainty to always be less than or equal to $22.00 during the investor's horizon, then the warrant would be worthless. This arises because the common stock price is less than exercise price.

Suppose the investor knew the stock price would rise to S = $27.00 prior to warrant expiration or investor horizon, whichever occurs first. Then the true value would be,

$$T.V. = 27 - 22 = 5.$$

Because the warrant price never falls below the true value, the premium that one must pay to play the game is just,

$$P = W - TV.$$

In terms of the stock/warrant diagram the premium is the vertical distance between the true value line and the current warrant price.

The limiting conditions imposed upon the warrant arise from arbitrage and a rational condition associated with any call feature. Arbitrage means to make two simultaneous transactions in securities such as to make a profit in the same security by buying and selling in different markets. In the case of warrants, suppose $W = 10$, $S = 40$, and $E = 22$. Then (ignoring commissions) one could purchase the warrant for \$10, pay the \$22 exercise price and obtain a share of the common stock for only \$32. Such a gift horse would not exist for any length of time. Thus all points to the right of the 45° segment of the true value line would create an arbitrage position because the warrant could be bought and exercised to effectively buy a share of stock below its current market price.

The rational condition of the call feature simply means that the warrant will never be worth more than the associated stock. On the stock/warrant diagram this is the warrant maximum value line. Thus, all points tend to fall between the two 45° lines. The stock/warrant diagram here shows the true value line, warrant maximum value line, and a typical regressed equation for the random variables, S, W, of TWA.

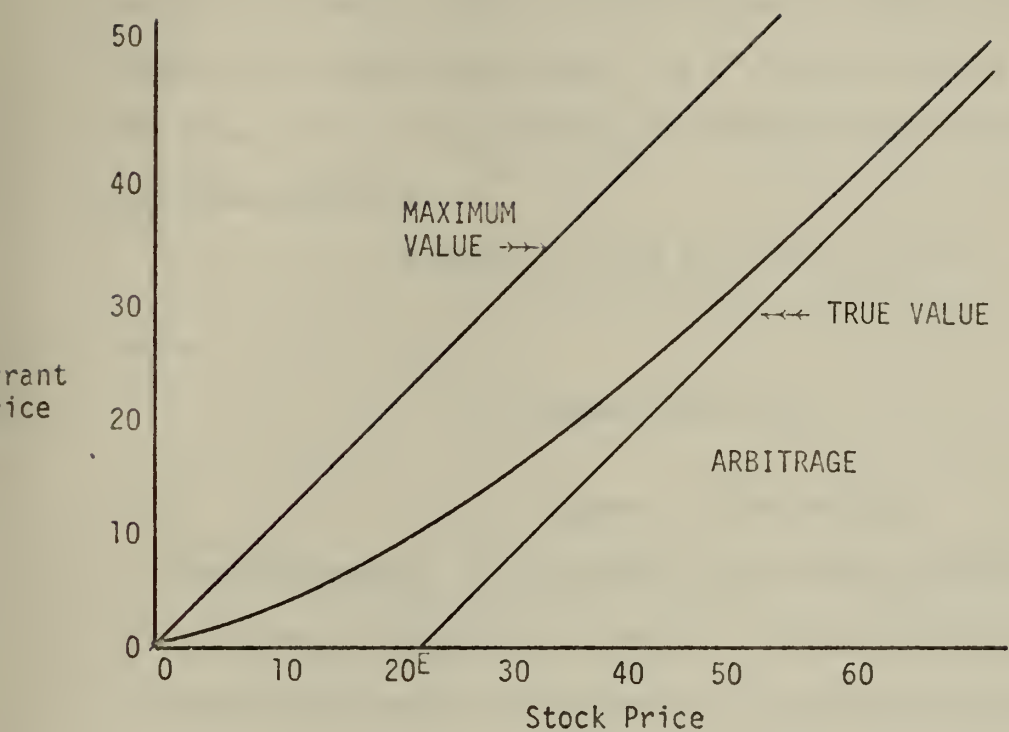

Figure 1. STOCK/WARRANT DIAGRAM

IV. THE HISTORICAL DEVELOPMENT OF WARRANT ANALYSIS

Scientific investigation of warrant price determination began early in the century with Bachelier (Ref. 3) in 1900, and later, Kruizenga (Ref. 13) while working under Samuelson, developed the "absolute difference" or arithmetic theory of Wiener Brownian motion for the option market. The conditional probability of the stock price at time τ given the price at t = 0 is a function of previous stock price differences over time period, τ.

$$PROB[S_{t+\tau} \leq s_\tau / S_o = s_o] = F(s_\tau - s_o, \tau)$$

where,

$$s_o = \text{Known price at } t_o$$

$$\tau = \text{Number of time periods}$$

The distribution, F, is deduced to be normal and the Wiener process has stationary, independent increments. A heuristic argument put forth to support this thesis is that prices are already discounted by previous knowledge of such things as earnings, growth, etc. One other problem concerns the possibility of the second moment of the random variable being infinite as sound in the Pareto-Levy class of distributions. This will not be discussed here.

The major weakness in the Bachelier thesis (Ref. 3) is that owners of stock possess limited liability which precludes that part of the normal density function argument with values less than zero, i.e., one cannot lose more than the investment worth such that s<o. To circumvent the difficulty, the simplest hypothesis is to postulate that each dollar's worth of common stock is subject to the same distribution, (Ref. 22, 23).

This means that precentage changes in stock prices are important to the investor rather than absolute price changes. This implies that the distribution at time $t+\tau$, given the price at time zero is a function of the ratio of the prices for τ time periods.

$$\text{PROB }[S_{t+\tau} \leq s_\tau | S_o = s_o] = F(\frac{s_\tau}{s_o};\tau)$$

If we assume by appealing to the central limit theorem that

$$\sum_{K=0}^{t+\tau-1} \text{LOG}(\frac{S_{k+1}}{S_k}) \sim N(\mu(t),\sigma^2 t),$$

where,

$$E[\text{LOG }\frac{S_{k+1}}{S_k}] = \mu_\tau$$

$$E[(\text{LOG}(\frac{S_{k+1}}{S_k} - \mu)^2] = \sigma^2 \quad .$$

Then since,

$$\sum_{K=0}^{t+\tau-1} \text{LOG}(\frac{S_{k+\tau}}{S_k}) = \sum_{K=0}^{t+\tau-1} [\text{LOG}(S_{k+\tau}) - \text{LOG}(S_k)]$$

$$= \text{LOG}(S_{t+\tau}) - \text{LOG}(S_o) \quad ,$$

which is also distributed normally as above.

Let

$$\frac{S_{k+1}}{S_k} = S'_k$$

$$P[\text{LOG}(S'_k) \leq \text{LOG }\frac{S_{t+1}}{S_t}] = \int_o^{\text{LOG}(\frac{s_{t+1}}{s_t})} \frac{1}{\sqrt{2\pi\sigma^2}} \text{ EXP}[-\frac{1}{2} \frac{(\text{LOG}(S'_k) - \mu)^2}{\sigma^2}] \, d[\text{LOG}(S'_k)] \; ,$$

since,

$$d[\text{LOG}(S'_k)] = \frac{1}{S'_k} dS'_k \; .$$

Then,

$$P[LOG(S'_k) \le LOG\frac{S'_{t+1}}{S_t}] = \int_{o}^{LOG(\frac{S_{t+1}}{S_t})} \frac{1}{S'_k\sqrt{2\pi\sigma^2}} \; EXP\,[-\frac{1}{2}\,\frac{(LOG(S'_k)-\mu)^2}{\sigma^2}]dS'_k \quad o<S'_k<\infty \,.$$

This is known as the log normal density.

Also note that,

$$E[S'_k] = \int_{o}^{\infty} e^{y} \,\frac{1}{\sqrt{2\pi\sigma^2}}\; EXP\,[-\frac{1}{2}\,\frac{(Y-\mu)^2}{\sigma^2}]d_y$$

$$= e^{\mu + \frac{1}{2}\sigma^2}$$

The following graph (Figure 2), comparing the normal density with mean of 0.0 and variance 0.5 with the log normal density with the same mean and variance shows how this interpretation eliminates the possibility of negative liability.

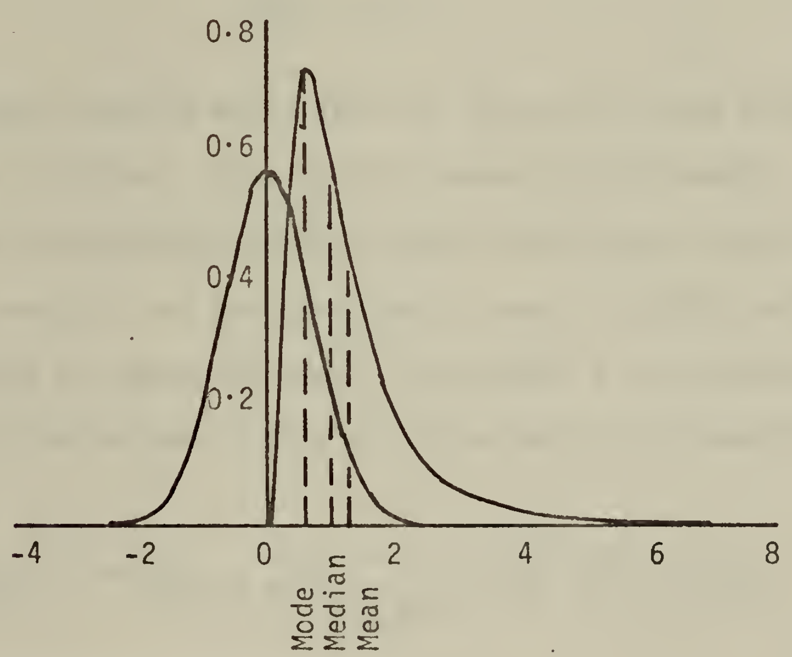

Figure 2. FREQUENCY CURVES OF THE NORMAL AND LOGNORMAL DISTRIBUTIONS.

Further treatment of the log normal is in Aitchison and Brown (Ref. 1).

It can be seen that for

$$\sum_{t=0}^{t} [\mathrm{LOG}(S_{t+1}) - \mathrm{LOG}(S_t)] = \mathrm{LOG} \prod_{t=0}^{\tau} S'_t \quad .$$

Thus,

$$E[\mathrm{LOG} \prod_{t=0}^{\tau} S'_t] = \mu\tau$$

$$E[\mathrm{LOG} \prod_{t=0}^{\tau} [S'_\tau - \mu]^2] = \sigma^2\tau$$

$$E[\prod_{t=0}^{\tau} S'_t] = e^{\tau(\mu+\frac{1}{2}\sigma^2)} \quad .$$

Samuelson (Refs. 22, 23) has applied the log normal distribution to warrant pricing. Further, he assumes that the mean or expected outcome of the stock will grow as compound interest, $\alpha \geq 0$.

$$E[\frac{S_\tau}{S_o}] = e^{\alpha\tau} \quad .$$

He further posits that the mean return on the warrant must equal the mean return on the stock. This at first appears unbelievable. But he argues that if this expected warrant return were higher than the expected stock return, would it not be discounted to equal the stock return?

Consequently the Samuelson model, establishes a relationship between stock price, S, the warrant's time to expiration, T, and exercise price, E, which is

$$W(S,T,\sigma^2,E,\alpha) = e^{-\alpha\tau} \int_o^\infty \mathrm{MAX}(o, s\ e^y - E) \frac{1}{\sigma\sqrt{2\pi T}} \ \mathrm{EXP}[-\frac{1}{2} \frac{(Y-T\mu)^2}{\sigma^2 T}]dy \quad .$$

After considerable manipulation,

$$W(S,T,\sigma^2,E,\alpha) = S\ N(V) - E\ e^{-\alpha T}\ N(V - \alpha\sqrt{T})$$

where,

$$V = [\mathrm{LOG}(\tfrac{S}{E}) + (\alpha+\tfrac{1}{2}\sigma^2)\ T]/\sigma\sqrt{T}\ .$$

Notice that as the time until expiration increases, as for a perpetual warrant, the expected warrant price approaches the stock price,

$$W(S,\infty,\sigma^2,E,\alpha) = S$$

These results may fail to occur in actuality because of the effects of stock and cash dividends, taxes, and transaction costs.

Notice also that for short T, and stock price equal to exercise price

$$V = (\alpha+\tfrac{1}{2}\sigma^2)\sqrt{T}/\sigma$$

$$S = E$$

and roughly,

$$N(\frac{\alpha\sqrt{T}}{\sigma} - \frac{1}{2}\sigma\sqrt{T}) = 0$$

so that,

$$W(E,T,\sigma^2,E,\alpha) \approx S\ N[(\alpha+\tfrac{1}{2}\sigma^2)\sqrt{T}/\sigma]$$

$$\approx K\sigma\sqrt{T}$$

$$K = \text{constant}$$

Further treatment of the application of the log normal to warrant pricing can be seen in Osborne (Ref. 20) and Boness (Ref. 5).

This equation is one of the more astounding results in price theory. It has been subjected to considerable testing. The reasonableness of the Samuelson conclusion that warrant price increases with the square root of time were conducted by Miller (Ref. 18). Miller stratified warrants into six categories according to time to expiration (Figure 3). His model for each stratum, T, is

$$W(s,T) = K\ S^b$$

where K and b are constants determined from a regression analysis. The coefficient, b, measures the elasticity of the warrant price. His final equations for the six stratum are:

Category (Time to Maturity)

1.	(6 months - 1 yr)	$W = .2753\ S^{1.8585}$	$R^2 = .96$
2.	(1 yr - 2 yr)	$W = .2547\ S^{1.6124}$	$R^2 = .94$
3.	(2 yr - 4 yr)	$W = .3703\ S^{1.4317}$	$R^2 = .8039$
4.	(4 yr - 7 yr)	$W = .4062\ S^{1.3095}$	$R^2 = .9582$
5.	(over 7 yr)	$W = .4263\ S^{1.1973}$	$R^2 = .8141$
6.	(perpetual)	$W = .5509\ S^{1.2155}$	$R^2 = .9926$

According to Samuelson,

$$W(E,T,\sigma^2,E,\alpha) \approx K\sigma\sqrt{T}\ .$$

For instance, a warrant with two years to expiration, S = E, would be worth, $\sqrt{2}$ = 1.4 or 40% more than one with one year to expiration.

Looking at Miller's work,

$$\frac{W(E,\ 1\ \text{yr} - 2\ \text{yr})}{W(E,\ 6\ \text{mo} - 1\ \text{yr})} = 1.29$$

$$\frac{W(E,\ 2\ \text{yr} - 4\ \text{yr})}{W(E,\ 1\ \text{yr} - 2\ \text{yr})} = 1.04$$

$$\frac{W(E,\ 4\ \text{yr} - 7\ \text{yr})}{W(E,\ 2\ \text{yr} - 4\ \text{yr})} = 1.10$$

$$\frac{W(E,\ \text{over}\ 7\ \text{yr})}{W(E,\ 4\ \text{yr} - 7\ \text{yr})} = 1.05$$

$$\frac{W(E,\ \text{perpetual})}{W(E,\ 4\ \text{yr} - 7\ \text{yr})} = 1.29$$

Although this table simplifies Miller's work somewhat, it does indicate that warrants with longer times to expiration are valued more highly than those with shorter times.

The following graph (Figure 4) shows the Miller model by category.

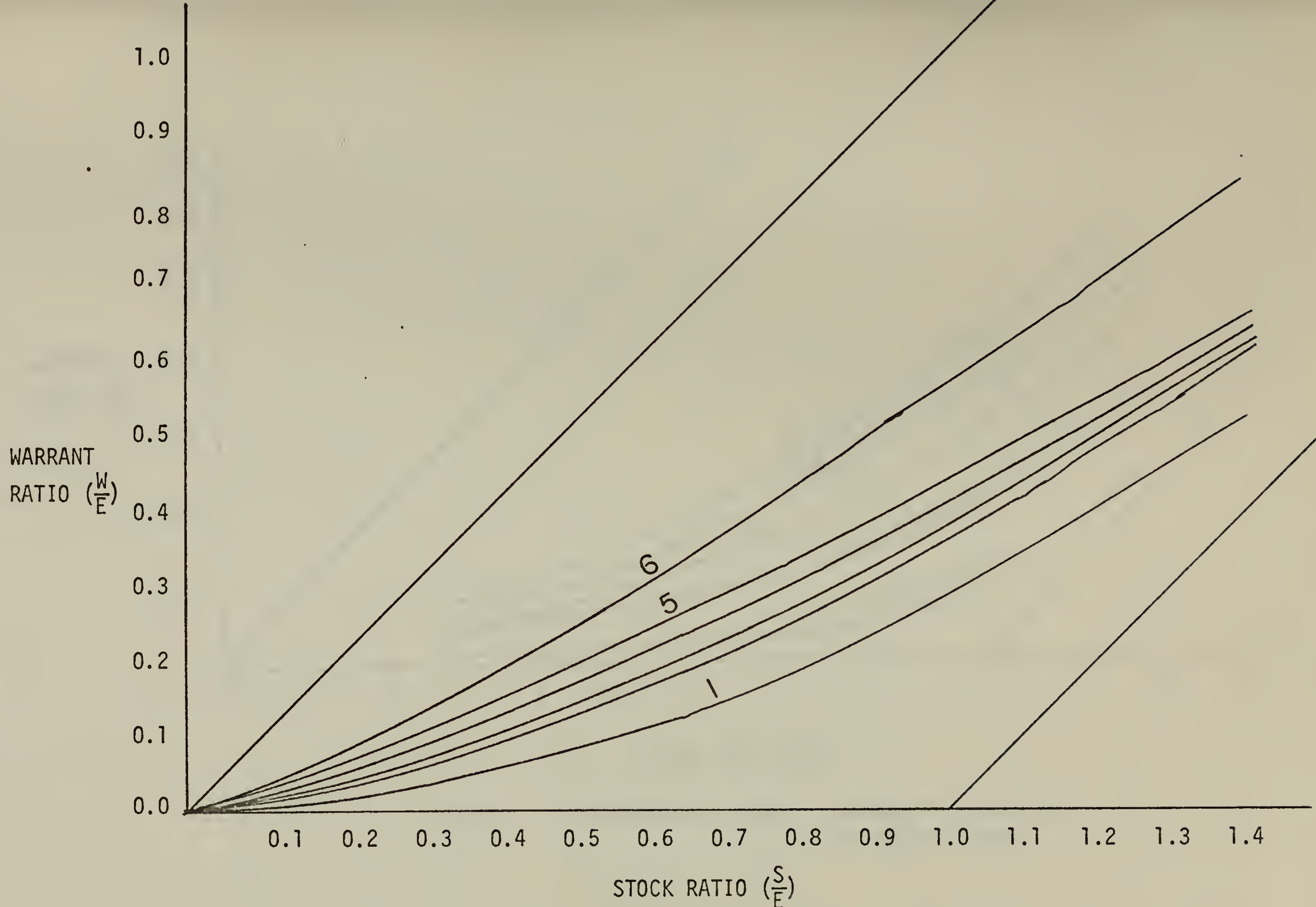

Figure 3. MILLER MODEL: LONGEVITY INFLUENCE ON WARRANT VALUE

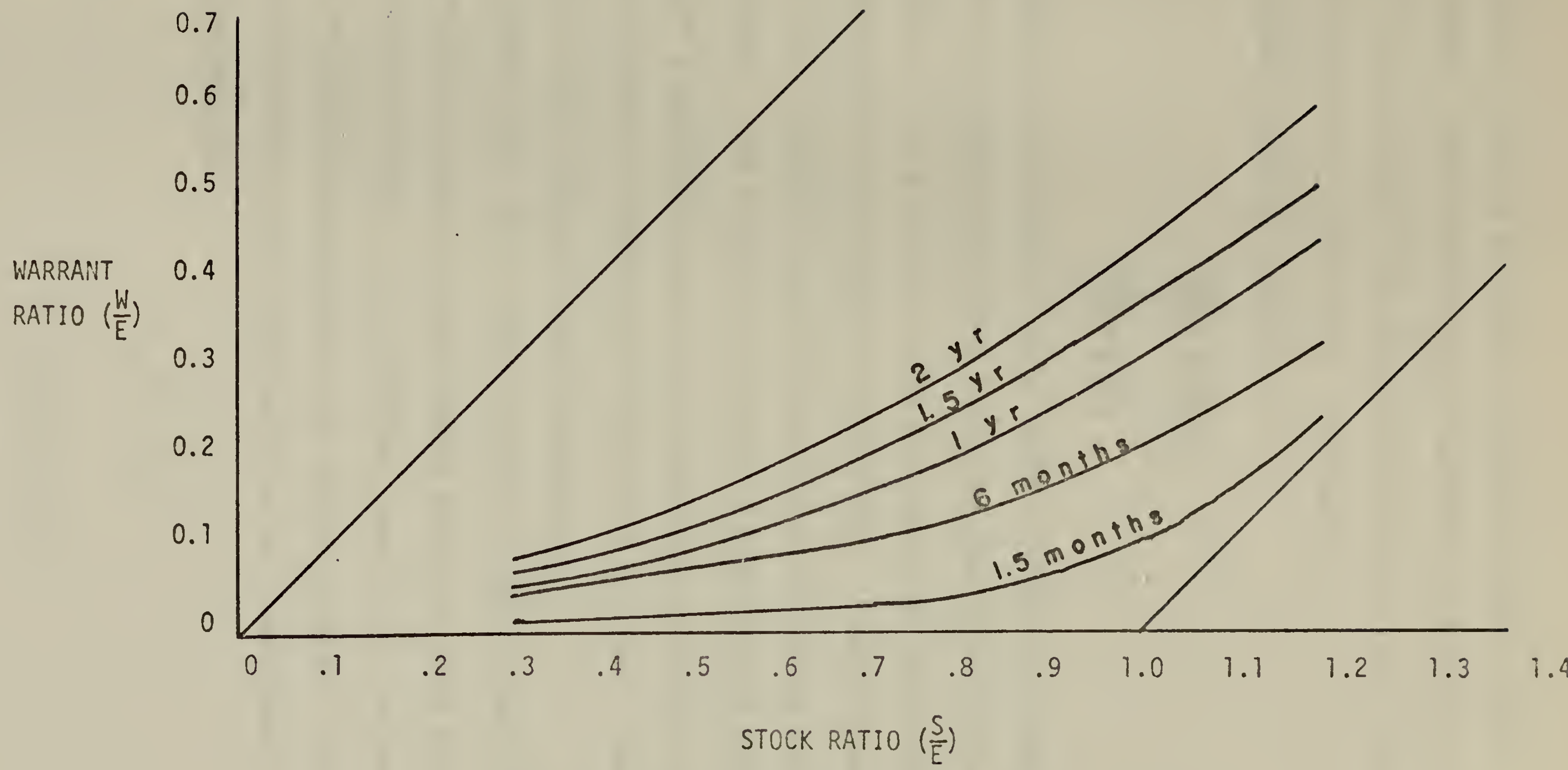

Figure 4. THORP-KASSOUF MODEL: LONGEVITY INFLUENCE ON WARRANT VALUE

Miller's results have been confirmed by other's work.

Another model developed by Kassouf (Ref. 12) and put into graphical form in a later work (Ref. 26) confirms that warrants with longer life times appear to be worth more. See Figure 4.

Comparing Kassouf's models at S = E

$$\frac{W(E,\ 1\ \text{yr})}{W(E,\ 6\ \text{months})} = 1.5$$

$$\frac{W(E,\ 2\ \text{yr})}{W(E,\ 1\ \text{yr})} = 1.43$$

These ratios do more closely resemble Samuelson's square root of time rule or 40% increase with twice the time to expiration.

Thus far the effects of stock price and time to expiration of warrants on the warrants' price have been discussed. It would appear that stock volatility, investor risk preference and utility could help in the analysis of warrants. The futility of trying to simultaneously measure marginal investor's expected values of stock volatility, utility, and risk preference because of underidentification will be shown. Sprenkle's investigations in this area will now be summarized.

Sprenkle (Ref. 24) in 1960 attempted to obtain quantitative measures of particular investors' mean expected change in stock price, the variance of these changes, and preferences toward risk. An example of the types of problems encountered in his analysis follows. Consider a warrant/stock with current price \$5/\$10 which can be represented as an invariant function over time.

$$W = f(S)$$

where

W = warrant price

S = stock price

Let this function be such that

$$2.5 = f(5); \quad 5 = f(10); \quad 10 = f(15).$$

Suppose an investor is indifferent to risk, i.e., linear homogeneous utility function, $U(x) = x$. Also the investor's expectations of stock price after time period t is,

$$S = \begin{cases} \$5 \text{ with probability } .35 \\ 10 \text{ with probability } .40 \\ 15 \text{ with probability } .25 \end{cases}$$

The investor's expectation of stock price after time period, t, is,

$$E(U(S)) = E(S) = 5 \times .35 + 10 \times .40 + 15 \times .25$$

$$= 9.50, \text{ or less than the current price of \$10.}$$

The same investor's expectation of warrant price is,

$$E(U(W)) = E(W) = 2.5 \times .35 + 5 \times .40 + 10 \times .25$$

$$= 5.375, \text{ or more than current price of \$5.}$$

Our risk indifferent investor might then buy the warrant rather than the stock since his $E(W)$ is greater than current price.

Now suppose our investor has a cubic utility function, $U(X) = X^3$ where X is the percentage gain on an investment, i.e., if S increases from \$10.00 to \$15.00 then $U(S) = (50\%)^3$. Let the warrant/stock relation remain the same, i.e., $f(5) = 2.5$, $f(10) = 5$, $f(15) = 10$. But now suppose the investor's expectations of stock price changes are:

$$S = \begin{cases} \$5 \text{ with probability } .25 \\ 10 \text{ with probability } .40 \\ 15 \text{ with probability } .35 \end{cases}$$

Then our new investor with utility function $U(X) = X^3$ will have expected warrant and stock values.

$$E(U(S)) = .25(-50\%)^3 + .40(0)^3 + .35(50\%)^3 = 12,500$$

$$E(U(W)) = .25(-50\%)^3 + .40(0)^3 + .35(100\%)^3 = 968,750$$

Once again our new investor will buy the warrant rather than the stock but for different reasons than in our first case. Our first investor was indifferent to risk and had lower stock price expectations than our second investor whose utility function was cubic.

Sprenkle (Ref. 25, p. 182) was unable to estimate all three parameters, expected price, variance, and risk preference. He estimated variances and whether investors were risk averters or not. In his analysis Sprenkle used as a proxy for risk a measure of leverage defined as,

$$L = \frac{dW/W}{dS/S} - 1 ,$$

where L is leverage per dollar of warrant and W is warrant price and S is stock price. To use this in his model he formulates the price an investor will pay for leverage as

$$[\frac{dW/W}{dS/S} - 1]W P_e$$

where P_e is the price an investor is willing to pay for one unit of leverage. Sprenkle conjectures that if $P_e > 0$ then either the investor believes the expected stock will be different from present price or the investor is not risk neutral.

Sprenkle's final model of the value, V, of the warrant is

$$V = E[W/S] + [\frac{dW/W}{dS/S} - 1]W P_e \quad .$$

Then assuming P_e is close to zero and the investor assumes stock prices log normally distributed,

$$V = \frac{K S}{\sqrt{2\pi}} \int_{B-\frac{\sigma}{2}}^{\infty} e^{-\frac{1}{2}\lambda^2} d\lambda - \frac{(1-P_e)E}{\sqrt{2\pi}} \int_{B+\frac{\sigma}{2}}^{\infty} e^{-\frac{1}{2}\lambda^2} d\lambda ,$$

where,

$$B = \frac{\text{Ln } E - \text{Ln } S - \text{Ln } K}{G}$$

S = stock price

E(S) = K S The investor thinks the expected price is some constant (K) times the present price.

In terms of the familiar stock/warrant diagram repeated below

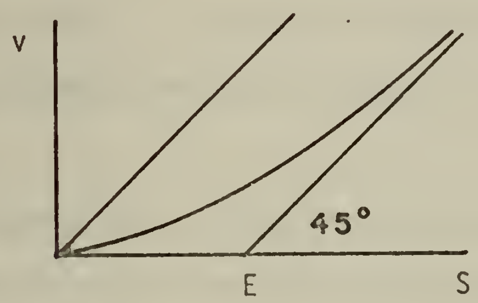

Figure 5. BASIC STOCK/WARRANT DIAGRAM

The diagram as modified by Sprenkle is,

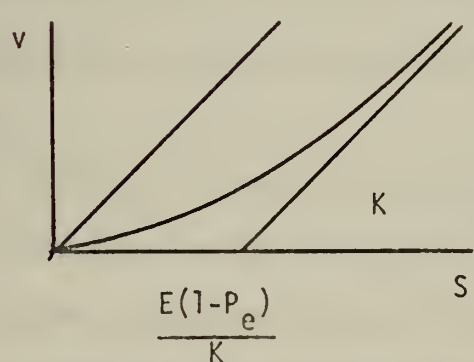

Figure 6. STOCK/WARRANT DIAGRAM AS MODIFIED BY SPRENKLE

Sprenkle only considered warrants with very long or infinite expriation dates (which elimates all short term warrants). Sprenkle was unable to estimate all three or even two of the parameters K, J, and P_e. Rather he assumed K = 1 and P_e = + .25, 0, or - .25. We are reminded that K reflects the investor belief of the direction of stock prices. Now if $P_e > 0$ either implication mentioned previously holds, i.e., (1) expected stock price is different from current price or (2) the investor is not risk neutral.

Sprenkle concludes that P_e is positive but the evidence is not clear due to the problem mentioned here as to whether an investor is risk averse or risk loving.

Another paper by Ayers (Ref. 2) indicates that warrant investors expect very high rates of returns on their investment. Ayers uses the log normal random walk as Sprenkle and results from Markowitz (Ref. 16) concerning portfolio selection. Boness (Ref. 5) developed a theory for call options that only considers options less than one year to live. This helps establish a time horizon which hampered Sprenkle. Boness assumed a log normal random walk and a risk neutral investor. The general conclusions that can be drawn here are the difficulty if not impossibility of trying to simultaneously estimate investor preferences (utility) and price expectations.

Van Horne (Ref. 27) used a little more simplistic model to evaluate warrants. He was interested in warrant price as a function of current available "safe" interest rates and the volatility of the associated stock. His study concerned a time series analysis of three perpetual warrants and their associated stocks. He concluded that the market price of a warrant varies directly with the value that an investor places

upon funds. As a proxy for value of funds the yield on 180 treasury notes was used. This same model attempted to use volatility as an explanatory variable, hypothesizing that the greater the volatility of the stock the larger the value of the warrant. As a proxy for volatility the coefficient of variation, $CV = \frac{\sigma}{\overline{P}_s}$, was used. $\overline{P}_s$ is the average market price of the stock over the previous 18 months and σ is the standard deviation about this price.

The important conclusion of his analysis is that warrant value increases with the value of funds and volatility of stock prices, (C.V.).

It should be noted that Van Horne (Ref. 28) erred in his second analysis of all warrants listed on the American Stock Exchange. This was a cross-sectional linear regression determining to what extent time, dividend, and a crude measure of volatility affected warrant price. He apparently oversimplified by not considering the consequences of using associated bonds at face value in lieu of cash. Several warrants listed in Reference 28 do have this option, i.e., Alleghany Corp, Braniff Airways, and Trans World Airlines.

In this section on the theory of warrant pricing an attempt has been made to show the following:

1) A general equilibrium model developed by Samuelson.

2) Evidence that time to expiration as developed by Samuelson is an important explanatory variable.

3) The underidentification problem associated with measuring investor preferences.

4) Current interest rates and associated stock variability do influence warrant price.

V. THE MODELS

The remainder of this paper summarizes the work of the author. A structural form is hypothesized in which time to expiration is discounted by considering only warrants with expiration times of seven or more years. This paradigm is then analyzed utilizing the Kassouf Model and a simple linear regression as a basis for comparison. Problems in the error terms of the Kassouf Model caused the analysis of this model to terminate while a small sample of warrants meeting specified criteria caused inconclusive evidence of results in judging the usefulness of the author's model.

It was decided to look at an empirical model which explained warrant price in terms of stock variability, investor risk preference, and warrant leverage. Time was not considered in an attempt to isolate the above three subjects. In each of these aspects a structure is specified and to the extent that the three in combination represent past performance future worth of the model is implied. Warrants used all have seven or more years to expiration, no associated bond option or increase in exercise price except for McCory Corporation which is noted.

The analysis centers about two stock warrant option price models. These models are predicated on the fact that stock price is known.

A. KASSOUF MODEL

The first model was devised by Sheen T. Kassouf, University of California, Irvine. (Ref. 12).

$$\left(\frac{W}{E}\right)_{\alpha,i} = \left[\left(\frac{S}{E}\right)_{\alpha,i}^{z_i} + 1\right]^{\frac{1}{z_i}} - 1 + \varepsilon_{\alpha,i} \quad i = 1,\ldots,4$$

where:

$$z = 1.307 + 5.355/t + 14.257\,R + .298\,D + 1.015\,\ln\left(\frac{S}{\bar{S}}\right) + .405\left(\frac{S}{E}\right)$$

t = time to expiration in months

R = dividend/S

S = stock price

$\bar{S}$ = mean of previous 11 month's high, low average

W = warrant price

E = exercise price

Notes:

1) Time to expiration is disregarded in that two of the warrants are perpetual and two are in excess of 84 months to expiration.

2) Since the derivation of the Kassouf model was not available an intuitive explanation is not included.

B. FITCH MODEL

The basis for the Fitch model rests upon investor attitude towards risk and stock price variability.

The following structural model was then specified.

$$\left(\frac{W}{E}\right)_{\alpha,i} = \left(\frac{S}{E}\right)_{\alpha,i} - 1 + \mathrm{EXP}\left[-\left(\frac{S}{E}\right)_{\alpha,i}[f(V)]^{-1}\right] + \varepsilon_{\alpha,i} \quad i = 1,\ldots,4$$

where:

$\frac{1}{E}$ serves to normalize S and W so that all stock/warrant combinations can be analyzed with the same model.

$f(v)$ is what will be used to describe volatility and expectation of the investor.

This specification was derived by noting the values of $\mathrm{EXP}(\frac{S}{E})$ as $(\frac{S}{E})$ varied $(0,\infty)$. Then to account for stock to stock differences the function $f(v)$ was introduced. Its inverse is taken so that $(\frac{W}{E})$ will increase as $f(v)$ increases.

Figure 7 is a sample realization of the four warrants considered and the two equations:

$$1)\quad \frac{W}{E} = \frac{S}{E} - 1 + e^{-\frac{1}{2}\left(\frac{S}{E}\right)}$$

$$2)\quad \frac{W}{E} = \frac{S}{E} - 1 + e^{-\left(\frac{S}{E}\right)} \,.$$

Warrants considered are:

* - Atlas Corp.

· - McCory Corp.

★ - Lerner Stores

⊙ - Alleghany Corp.

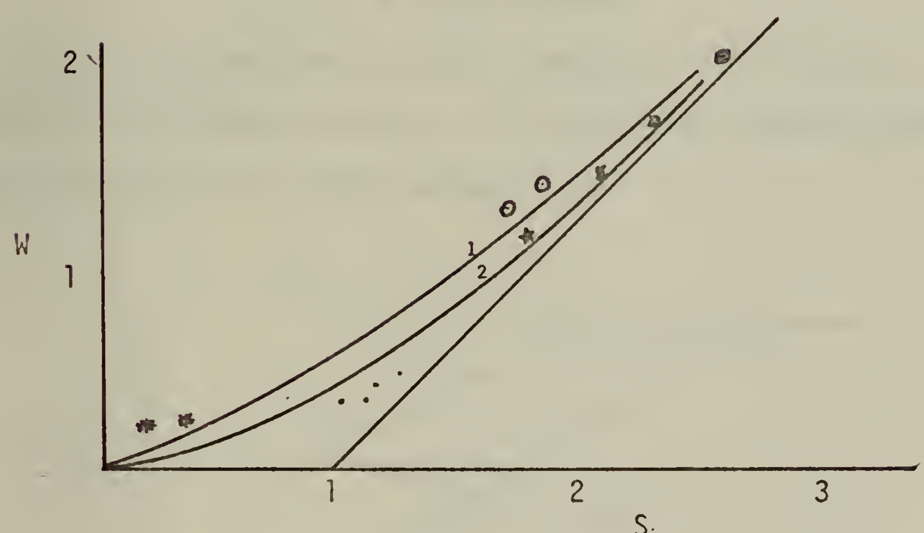

Figure 7. SAMPLE REALIZATION OF DATA COLLECTION

Further, the Fitch model has heuristic appeal in the context of utility theory. It will be shown that a preference ordering which is independent of decision maker wealth can be represented as,

$$U(x) = a + be^{\alpha x}$$

where,

$$x = \text{total assets}$$

This is similar to the structure of the Fitch model. An intuitive argument for wealth independence can be based upon the heterogeneous character of investors in the stock market, i.e., large institutional investors as well as the small investor with much less total assets.

To see that $U(x) = a + b\ e^{\alpha x}$ implies wealth independence it is desirable to define risk as,

$$r(x) = -\frac{U''(x)}{U'(x)}$$

where,

$$x = \text{total assets}$$

It will be shown that if $r(x)$ = constant, then $U(x)$ is a wealth independent preference ordering. The following diagram (Figure 8) is a utility function for a risk averter.

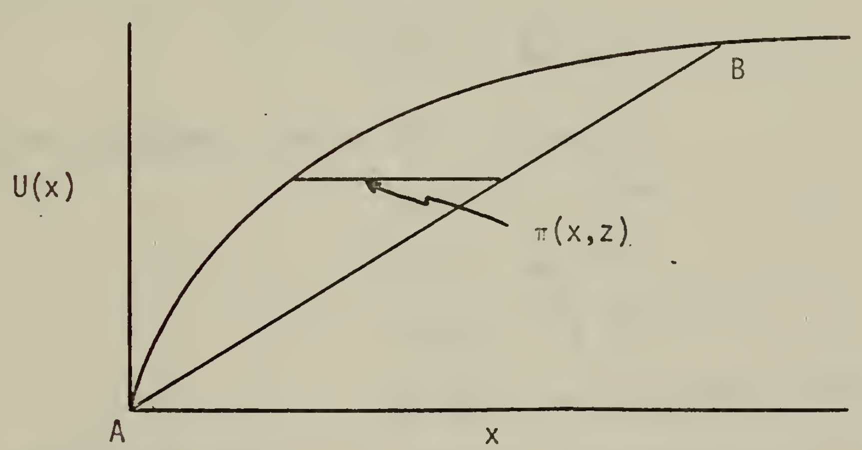

Figure 8. UTILITY CURVE (RISK AVOIDER): MEASURING RISK

Given assets, x, define a risk premium, $\pi(x,z)$, such that one with utility $U(x)$, is indifferent between a gamble, $z = 0p + (1-p)x$, $0 \leq p \leq 1$, and a non-random return, $E(z) - \pi(x,z)$. The line AB represents the gamble, z. It can now be said that,

$$E(U(z)) = U(E(Z)-\pi(x,z))$$

and at any level of assets, x,

$$E(U(x+z)) = U(x+E(z)-\pi(x,z)) ,$$

Then by scaling, U, such that $E(z) = 0$,

then, $$E(U(x+z)) = U(x-\pi(x,z)).$$

Let $$Var(z) = \sigma^2,$$

using Taylor expansion,

$$U(x-\pi(x,z)) = U(x)-\pi(x,z)U'(x)+\mathcal{O}(\pi^2)$$

and $$E(U(x+z)) = E(U(x)+zU'(x)+1/2z^2U''(x)+\mathcal{O}(z^3)).$$

Combining the two equations above, we get,

$$\pi(x,z) = -1/2\sigma^2 \frac{U''(x)}{U'(x)} +\mathcal{O}(\pi^2,z^3).$$

If the last term is neglected then,

$$\pi(x,z) = \frac{\sigma^2}{2} r(x) .$$

Using the structural form of the Fitch model,

$$U(x) = a + b\, e^{\alpha x}$$

$$U'(x) = \alpha b\, e^{\alpha x}$$

$$U''(x) = \alpha^2 b\, e^{\alpha x}$$

$$\frac{-U''(x)}{U'(x)} = -\alpha, \text{ a constant risk premium,}$$

which implies the model is of a wealth independent preference ordering.

In order to use the computer program of Prof. John Tukey, Princeton University, called SNAP/IEDA, which uses step wise regression, the models were:

1) KASSOUF: z was computed using the coefficients as given. Then, $[(\frac{S}{E})_{\alpha,i}^{z_i} + 1]^{\frac{1}{z_i}}$, was computed and regressed against $(\frac{W}{E})_{\alpha,i}$.

2) FITCH:

$$\frac{-\left(\frac{S}{E}\right)_{\alpha,i}}{\ln[(\frac{W}{E})_{\alpha,i} - (\frac{S}{E})_{\alpha,i} + 1.002]} = f(v) + \varepsilon_{\alpha,i} \quad i = 1,\dots,4$$

where,

$$f(v) = \beta_o + \beta_1 \hat{L}_{i,\alpha-1} + \beta_2 Y_{i,\alpha-1} + \beta_3 D_i + \beta_4 R_i$$

$$\hat{L}_{i,\alpha-1} = \frac{\mathrm{MAX}[(\frac{S}{E})_{i,\alpha-1} - 1,0]}{(\frac{W}{E})_{i,\alpha-1}}$$

$$Y_{i,\alpha-1} = \frac{\mathrm{DIVIDEND}}{S_{i,\alpha-1}}$$

$$D_i = \frac{\text{\# shares warrant can call}}{\text{total outstanding shares}}$$

$$R_i = \text{range of stock price 1971}$$

Notes:

1) $\hat{L}$ is the typical brokerage house method of defining leverage.

2) This is a first order autoregressive model.

3) The number 1.002 was used to replace 1.0 as previously specified to keep $\ln(0) = -\infty$ from occurring. Certain data points were on the true value line.

VI. DATA COLLECTION

The C&P Research, Inc. News Letter (Ref. 9) was used as a clearinghouse in choosing warrants. It was decided to focus attention on the period January-December 1972. This was a period of generally rising prices throughout.

The following are end of year data, 1972 for the four stock warrant combinations considered.

	S	W	E	YIELD	1972 STOCK RANGE
Alleghany Corp.	10.88	7.25	3.75	2.6%	0.40
Atlas Corp.	2.13	1.13	6.25	0	0.68
Lerner Stores	41.25	26.88	15.00	2.3	0.37
McCory Corp.	22.75	10.13	20.00	5.3	0.35

Notes:

1) All warrants have four or more years to run.

2) Stock dividend yields are relatively low in that no mortgage co's or real estate investment trusts with 9-10% yields are considered.

3) Price range is the 1972 hi-lo difference divided by end of year closing prices. For the models this was replaced by 1971 range.

4) Barrons newspaper was used with one set of data taken each week. There were 45 observations per stock for a total of 180 data points.

5) McCory does have a step up exercise price March 15, 1976 to $22.50. This was thought to be inconsequential during the time considered.

VII. THE ANALYSIS

Both the Kassouf and the Fitch models are evaluated in light of a simple linear model involving the same explanatory variables.

$$\left(\frac{W}{E}\right)_{\alpha,i} = \beta_o + \beta_1\left(\frac{S}{E}\right)_{\alpha,i} + \beta_2 Y_{\alpha-1} + \beta_3 D_{\alpha-1} + \beta_4 R_i + \varepsilon_{\alpha,i}$$

Under asymptotic distribution theory, consistency of least squares requires that $E[\varepsilon|X] = 0$ and that VAR $[\varepsilon|X] = \sigma^2$, where $\sigma^2 < \infty$, x is a matrix of independant variables, and y is a vector of dependant variables. In this case, $b = (X'X)^{-1} X'y$ is a consistent estimator of β if $(\frac{1}{n})X'X$ converges to a positive definite matrix as $n \to \infty$. Also, $S^2 + e'e/(n-k)$ is a consistent estimator of σ^2 if the distribution of y given X is n-variate normal (Ref. 25, p. 362).

A. HETEROSCEDASTICITY

Under the assumption of the standard linear model and ε_1~iid $N(0,\sigma^2)$ a test for heteroscedasticity can be performed as follows:

$$H_o: \ \mathrm{VAR}(\varepsilon_i) = I \qquad 1 = 1,2$$

$$H_A: \ \mathrm{VAR}(\varepsilon_i) \neq I$$

utilizing the test statistic,

$$\frac{e'_1 e_1}{e'_2 e_2} \sim F\left(\frac{n}{2} - K\right) .$$

In light of the Samuelson argument to use the log normal distribution, the stock prices were split into two groups of lower and higher stock price. The critical value of F at 1% level of significance is $F_{(75,80)}$ = 1.75. 1.75 is at a lessor number of degrees of freedom for the numerator

then tested so the level of significance would actually be somewhat greater.

The computed values are

MODEL	F COMPUTED
linear	1.86
Kassouf	4.36
Fitch	7.7

Thus in each case the null hypothesis is rejected.

Utilizing Atken's Estimator where we assume that linearity applies and that $\nu(y/x) = \sigma^2 V$ $(\sigma^2 < \alpha)$, V is a symmetric positive definite matrix we can transform the models with $X'P'Py = X'P'PX\,\hat{\beta}$, where $P'P = V^{-1}$ and $\hat{\beta} = (x'v^{-1}x)^{-1}\,x'v^{-1}y$ is the generalized least squares estimator of β, (Ref. 25, p. 238).

In this case the models appear as,

$$\left(\frac{W}{S}\right)_{\alpha,i} = \frac{\beta_o}{\left(\frac{S}{E}\right)_{\alpha,i}} + \beta_1 + \beta_2\frac{Y_{\alpha-1,i}}{\left(\frac{S}{E}\right)_{\alpha,i}} + \beta_3\frac{D_i}{\left(\frac{S}{E}\right)_{\alpha,i}} + \beta_4\frac{R_i}{\left(\frac{S}{E}\right)_{\alpha,i}} + \frac{\varepsilon_{\alpha,i}}{\left(\frac{S}{E}\right)_{\alpha,i}}$$

$$\left(\frac{W}{E}\right)_{\alpha,i} = \frac{\left(\left(\frac{S}{E}\right)_{\alpha,i} + 1\right)^{\frac{1}{zi}}}{\left(\frac{S}{E}\right)_{\alpha,i}} - \frac{1}{\left(\frac{S}{E}\right)_{\alpha,i}} + \frac{\varepsilon_{\alpha,i}}{\left(\frac{S}{E}\right)_{\alpha,i}}$$

$$\frac{-1}{\ln\left[\left(\frac{W}{E}\right)_{\alpha,i} - \left(\frac{S}{E}\right)_{\alpha,i} + 1.002\right]} = \frac{1}{\left(\frac{S}{E}\right)_{\alpha,i}}\left[\beta_o + \beta_1\,\hat{L}_{\alpha-1,i}\right.$$

$$\left. + \beta_2\,Y_{\alpha-1,i} + \beta_3\,D_i + \beta_4\,R_i + \varepsilon_{\alpha,i}\right]$$

Stepwise regression was then performed on these models. The results before and after transformation are as follows.

Linear Model

Before:

STD. ERROR .065

$$\left(\frac{W}{E}\right)_{\alpha,1i} = 0.07 + \underset{(.014)}{0.782}\left(\frac{S}{E}\right)_{\alpha,i} - 3.287Y_{\alpha-1,i} - \underset{(.08)}{0.414}D_i - \underset{(.001)}{0.01}R_i$$

After, and rearranging terms to coincide to the above

$$\left(\frac{W}{E}\right)_{\alpha,i} = 0.76 + \left(\frac{S}{E}\right)_{\alpha,i} - \underset{(.191)}{7.147}\, Y_{\alpha-1,i} - \underset{(.008)}{0.821}\, D_i$$

STD. ERROR (Computed from $\sqrt{\frac{\sum_{i=1}^{n} (Y_i-\hat{Y}_i)^2}{n}}$)
.07

Kassouf Model

Before:

STD. ERROR .093

$$\left(\frac{W}{E}\right)_{\alpha,i} = -1.016 + \underset{(.007)}{1.023}\left[\left(\frac{S}{E}\right)_{\alpha,i}^{zi} + 1\right]^{\frac{1}{z_i}}$$

After:

$$\left(\frac{W}{E}\right)_{\alpha,i} = -1.234 - \underset{(.023)}{0.27}\left(\frac{S}{E}\right)_{\alpha,i} + 1.364\left[\left(\frac{S}{E}\right)_{\alpha,i}^{z_i} + 1\right]^{\frac{1}{z_i}}$$

STD. ERROR .037

Fitch Model

Before:

$$\left(\frac{W}{E}\right)_{\alpha,i} = -.953 + \underset{(.007)}{.981}\left(\frac{S}{E}\right)_{\alpha,i} + \underset{(.026)}{.939}\, \mathrm{EXP}\left[\left(-\left(\frac{S}{E}\right)_{\alpha,i}\right)f(v)^{-1}\right]$$

STD. ERROR .067

$$f(v) = 3.060 - \underset{(.38)}{.44}\, \hat{L}_{\alpha,i} - \underset{(30.671)}{38.352}Y_i - \underset{(.892)}{2.527}\, D_i - \underset{(.023)}{.055}\, R_i$$

After:

$$\left(\frac{W}{E}\right)_{\alpha,i} = -1.003 + \underset{(.005)}{.911}\left(\frac{S}{E}\right)_{\alpha,i} + \underset{(.030)}{1.081}\, \mathrm{EXP}\left[\left(-\left(\frac{S}{E}\right)_{\alpha,i}\right) f(v)^{-1}\right]$$

STD. ERROR .068

$$f(v) = 4.339 + \underset{(12.010)}{12.753}\, Y_{\alpha-1} - \underset{(1.216)}{1.887}\, D_i - \underset{(.055)}{.248}\, R_i$$

Standard errors of the coefficients are reported in brackets below the coefficient.

After the Fitch model was regressed as on the previous page it was then rearranged as it appears above. The standard errors are for the model as it appears above.

In both the linear model and Fitch model the standard error increased. More will be said of the Kassouf model later. This is reasonable from the characteristics of GLS standpoint since it is the residual vector after transformation that is being minimized and not residual vector before transformation. The coefficients of L, $Y_{\alpha-1,i}$, D_i all appear insignificant at the 95% level using the t-test statistic in the Fitch model. Thus one may draw inferences about the usefulness of explanatory variables other than $\frac{S}{E}$.

B. AUTOCORRELATION

In time series analysis it is frequently unrealistic to assume that disturbances are uncorrelated. If disturbances are first order autoregressive we can say that,

$$\varepsilon_\alpha = \rho\, \varepsilon_{\alpha-1} + \zeta_\alpha$$

where,

$$|\rho| = |E(\varepsilon_\alpha, \varepsilon_{\alpha-1})| < 1$$

process describes the situation where,

$$\zeta_\alpha \sim N(o, \sigma_o^{\ 2})$$

and is called white noise.

In the case of the linear model and the Kassouf model the Durbin Watson statistic was computed to test for first order autocorrelation of disturbances as outlined in Ref. 25 (p. 199).

			tabled values		
	d	4-d	d_l	d_u	
linear	1.86	2.14	1.57	1.78	(n=100,k=6)
Kassouf	1.00	3.00	1.65	1.69	(n=100,k=2)

For the linear model the null hypothesis that the error terms are not autocorrelated (first order) cannot be rejected at the 95% level. However, the Kassouf model null hypothesis can be rejected in favor of the alternate that there is positive autocorrelation.

The Fitch model is unique in that it is a first order autoregressive model in that warrant price (the dependent variable) appears as a lagged explanatory variable in the leverage function, $\hat{L}$. Ref. 25 (p. 414) states that the estimate of β under conditions of an autoregressive function based upon L.S. is not consistent.

$$\lim_{n\to\infty} P\left[\left|\hat{\rho}_n - \rho\,\frac{\beta(\beta+\rho)}{1+\beta\rho}\right|>\varepsilon\right] = 0 \quad \text{for any } \varepsilon>o$$

And that this value is biased towards zero suggesting that the Durbin Watson statistic is asymptotically biased towards acceptance of the null hypothesis. The power of the Durbin-Watson test being reduced with decreased sample size. However, theorem 8.5 states that

$$b = (x'x)^{-1}xy$$

$$S^2 = \frac{e'e}{(n-k)}$$

are consistent for β,σ^2 respectively if:

1) All G roots of $Z^G - \beta_i Z^{G-1} - \beta_2 Z^{G-2} - .. - \beta_G = 0$ are $|\sigma|<1$

2) Convergence of moment matrices of fixed variables.

Taking the Fitch model as an autoregressive form,

$$\frac{(S/E)_{\alpha,i}}{\mathrm{LOG}[(\frac{W}{E})_{\alpha,i} - (\frac{S}{E})_{\alpha,i} + 1.002]} = 3.06 - .44\,\hat{L}_{\alpha-1,i} - 38.352\,Y_i - 2.527\,D_i$$

$$- .55\,R_i + \varepsilon_{\alpha,i}$$

our only concern is the term, $\hat{L}_{\alpha-1,i}$ which contains $(\frac{W}{E})_{\alpha-1,i}$.

Disregarding the terms not associated with autoregression and looking at the form of the equation,

$$Y_{\alpha,i} = -.44\, Y_{\alpha-1,i} + \varepsilon_{\alpha,i} + C \quad ,$$

$$C = \text{constant for other terms}$$

defining $B(y_\alpha) = y_{\alpha-1}$ as the backward transfer fn.

We can write

$$C + \varepsilon_\alpha = y_\alpha(1 - .44B)$$

$$y_\alpha = \frac{\varepsilon_\alpha + C}{1-.44B}$$

$$= \sum_{j=o}^{\infty} (.44B)^j[\varepsilon_\alpha + C]$$

which converges for all $B \leq 1$. So that we can justify the use of theorem 8.5 and subsequently accept the validity of the Durbin Watson test.

	d	$4-d$	d_l (k=6,n=100)	d_u
Fitch model	1.75	2.25	1.57	1.75

C. NORMALITY

In order to determine if errors were normally distributed a chi square goodness of fit test was utilized. The critical value for the chi square with 120 df (maximum tabulated) was 140.23 at the 90% level. All models have been corrected for heteroscedasticity.

Computed Values:

linear	19.19	8 equally likely int.
Kassouf	8.353	8 equally likely int.
Fitch	16.53	8 equally likely int.

Thus the null hypothesis, that the $\varepsilon_i \sim N(o,\sigma^2_i)$ cannot be rejected.

VIII. CONCLUSIONS

A. The Kassouf model is both heteroscedastic and first order autocorrelated. It was decided that one or the other problems could be alleviated but not both simultaneously without changing the functional form. Since the motives for its use are not known, this was not done.

B. Of the other two models the linear model is ranked better in terms of standard error.

	S.E.
linear	.070
Fitch	.076

C. Utilizing the t statistic for coefficients in the Fitch model, the only coefficients significantly greater than zero are the constant term and range. Notice that $\hat{L}$, the brokerage leverage indicator is insignificant.

D. Without further evidence the linear model is considered best.

E. It is felt that a more representative sample of warrant options may significantly change the results given here. Of the four warrants considered, Atlas is extremely nonvolatile, moving on the average 1/4 of a point a day within the range (2-3). The other three stocks were located near the minimum value arbitrage line. None had stock prices which ranged (less than 1, greater than 1) on the $\frac{S}{E}$ scale, (See Figure 8). This would have yielded information about investor preference with the stock near its exercising price.

F. It may also be useful to redefine leverage as the coefficient of variation, $\frac{\sigma_s}{\bar{P}_s}$ as in the Miller model.

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The following is an analysis of stock option warrants from the investor point of view. A survey of the literature presents the forms of analysis used to date. A model is called the Fitch Model, for reasons of humility, which will explain warrant value in terms of associated stock variability, yield, leverage, and potential common stock dilution. Time to expiration of the warrant is discounted by considering only warrants with more than

20. Abstract Cont

seven years until expiration. The analysis also presents two other models, Kassouf's and a linear regression, as a basis for comparison. The conclusions are that the Kassouf model is both heteroscedastic and first order autocorrelated and could not support further analysis without modifying its structure. Of the two remaining models the linear model yields superior predictions as measured by its standard error. It is also felt that a more representative sample of warrants may significantly change the results given here.

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